Application of CCRD to modelling the effect of variables on the performance of the 3-product cyclone

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Abstract

Earlier work showed that the length of the second vortex finder had a substantial influence on the performance of the three-product cyclone. However, the statistical significance of this effect was not known. To determine the significance and model this effect, and those of the diameter of the second vortex finder, cyclone feed percent solids and inlet pressure, a process analysis and modelling technique that requires fewer tests and gives almost as much information as a three-level factorial design, namely the CCRD, was used.

The effects of the length and diameter of the second vortex finder, and the cyclone feed percent solids were found to be significant in most cases. The resultant model is also significant, predicts the experimental data well and can be used to estimate the response corresponding to operating conditions not included, but fall within the range of conditions in the experimental design.

Keywords: Classification; comminution; hydrocyclones; three-product cyclone, modelling

1. INTRODUCTION

The design, separation mechanism and some potential applications of the three-product cyclone have been described in detail by Obeng and Morrell (2003). The main features of the cyclone are depicted in Figure 1. As can be seen, the unit is a conventional hydrocyclone with a modified top cover plate and a second vortex finder inserted so as to generate three product streams - an Inner Overflow (INO), an Outer Overflow (OUO) and an underflow. The conventional and second vortex finders are referred to as Outer Vortex Finder (OVF) and Inner Vortex Finder (IVF) respectively. The unit uses a smaller spigot size than the conventional cyclone in order to create crowding and hindered settling conditions in the conical section. With the IVF length extending well into the cylindro-conical body, an additional exit is provided for the crowded particles. Hence the potential for underflow roping is reduced.

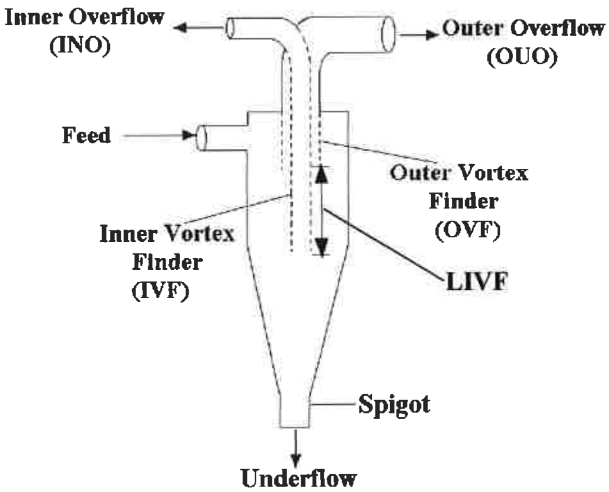

Figure 1. Main features of the three-product cyclone

Work by Obeng and Morrell (2003) showed that the length of the second vortex finder had a substantial influence on the operational performance of the three-product cyclone. However, the statistical significance of this effect was not known. To determine the significance and model this effect, and those of the diameter of the second vortex finder, feed percent solids and inlet pressure, an appropriate experimental design technique had to be used.

The experimental design techniques commonly used for process analysis and modelling are the full factorial, partial factorial and central composite rotatable designs. A full factorial design requires at least three levels per variable to estimate the coefficients of the quadratic terms in the response model. Thus for the four independent variables mentioned above, 3^4 or 81 experiments, plus replications would have to be conducted. It has also been shown that a 3^k factorial design estimates the coefficients of the squared terms in the model with relatively low precision (Box and Wilson, 1951). A partial factorial design requires fewer experiments than the full factorial. However, the former is particularly useful if certain variables are already known to show no interaction (Box and Hunter, 1961).

An effective alternative to the factorial design is the central composite rotatable design (CCRD), originally developed by Box and Wilson (1951) and improved upon by Box and Hunter (1957). The CCRD gives almost as much information as a three-level factorial, requires much fewer tests than the full factorial and has been shown to be sufficient to describe the majority of steady-state process responses (Cilliers et al., 1992; Crozier, 1992).

In this paper, the requirements for the CCRD and its application to the design of experiments, significance testing and modelling the effect of the four variables on the performance of the three-product cyclone are described.

2. Requirements for the CCRD

The number of tests required for the CCRD includes the standard 2^k factorial with its origin at the centre, 2k points fixed axially at a distance, say β, from the centre to generate the quadratic terms, and at least one test at the centre; where k is the number of variables. The axial points are often chosen such that they allow rotatability (Box and Hunter, 1957) which ensures that the variance of the model prediction is constant at all points equidistant from the design centre. Replicates of the test at the centre are very important as they provide an independent estimate of the experimental error. For four variables, the recommended number of tests at the centre is six (Box and Hunter, 1957). Hence the total number of tests required for the four independent variables is $2^4 + (2 \times 4) + 6 = 30$, which is at least, 51 experiments less than that required for a full factorial design. Figure 2 shows the CCRD and the co-ordinates for k = 4 factors.

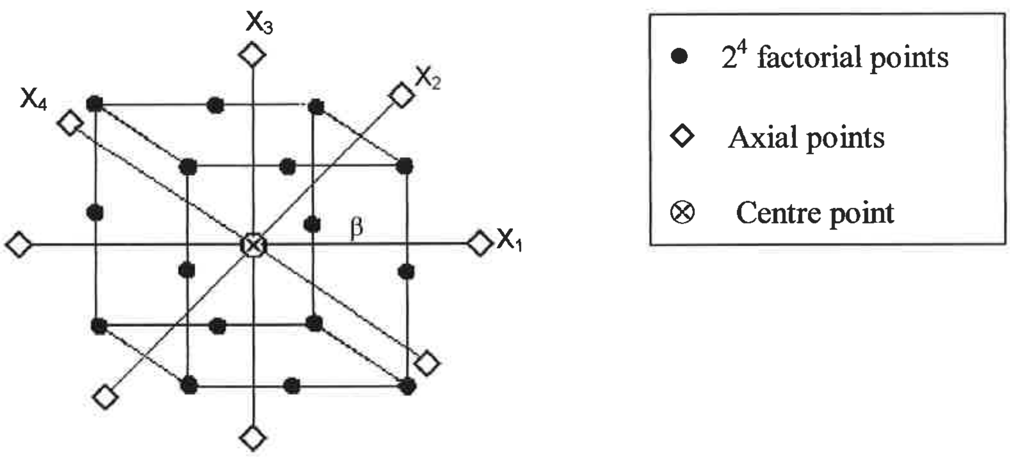

Figure 2. A CCRD for four factors x_1, x_2, x_3 and x_4

Once the desired range of values of the variables are defined, they are coded to lie at ±1 for the factorial points, (0,0) for the centre points and ±β for the axial points. The codes are calculated as shown in Table 1.

Table 1 - Relationship between coded and actual values of a variable (Napier-Munn, 2000)

Code	Actual value of variable
$-\beta$	x_{min}
-1	$\frac{(x_{max}+x_{min})}{2} - \frac{(x_{max}-x_{min})}{2\alpha}$
0	$\frac{(x_{max}+x_{min})}{2}$
$+1$	$\frac{(x_{max}+x_{min})}{2} + \frac{(x_{max}-x_{min})}{2\alpha}$
$+\beta$	x_{max}

x_{max} and x_{min} = maximum and minimum values of x respectively; $\alpha = 2^{k/4}$; k = number of variables

When the response data are obtained from the test work, regression analysis is carried out to determine the coefficients of the response model (a_1, a_2, ..., a_n), their standard errors and significance. In addition to the constant (a_0) and error (ε) terms, the response model incorporates:

- Linear terms in each of the variables (x_1, x_2, ..., x_n)
- Squared terms in each of the variables (x_1^2, x_2^2, ...x_n^2)
- First order interaction terms for each paired combination (x_1x_2, x_1x_3, ..., $x_{n-i}x_n$)

Thus for the four variables under consideration, the response model is:

$$a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + a_{44}x_4^2 + a_{12}x_1x_2 + a_{13}x_1x_3 + a_{14}x_1x_4 + a_{23}x_2x_3 + a_{24}x_2x_4 + a_{34}x_3x_4 + \varepsilon \quad (1)$$

A detailed analysis of variance (ANOVA) is also carried out to determine the statistical significance of the linear, square and interaction terms in the response model.

3. Experimental design

To provide data to determine the statistical significance and model the effect of the variables on the performance of the three-product cyclone via the CCRD approach, the range of values for each variable was defined as follows:
Length of IVF below the OVF (LIVF): 50 – 585 mm
Diameter of IVF (DIVF): 35 – 50 mm
Cyclone feed percent solids: 30 – 60 %
Cyclone inlet pressure: 80 – 130 kPa

Using the relationships in Table 1, the actual levels of the variables for each of the thirty experiments in the design matrix were calculated. Table 2 gives the coded and actual levels of the variables.

Table 2 - Coded and actual levels of variables

	Test No	Coded level of variables: Pressure	% Solids	LIVF	DIVF	Actual levels of variables: Pressure (kPa)	% Solids (w/w)	LIVF (mm)	DIVF (mm)
Factorial Points	1	-1	-1	-1	-1	92.5	37.5	183.8	38.8
	2	+1	-1	-1	-1	117.5	37.5	183.8	38.8
	3	-1	+1	-1	-1	92.5	52.5	183.8	38.8
	4	-1	-1	+1	-1	92.5	37.5	451.3	38.8
	5	-1	-1	-1	+1	92.5	37.5	183.8	46.3
	6	+1	+1	-1	-1	117.5	52.5	183.8	38.8
	7	+1	-1	+1	-1	117.5	37.5	451.3	38.8
	8	+1	-1	-1	+1	117.5	37.5	183.8	46.3
	9	-1	+1	+1	-1	92.5	52.5	451.3	38.8
	10	-1	+1	-1	+1	92.5	52.5	183.8	46.3
	11	-1	-1	+1	+1	92.5	37.5	451.3	46.3
	12	+1	+1	+1	-1	117.5	52.5	451.3	38.8
	13	+1	+1	-1	+1	117.5	52.5	183.8	46.3
	14	+1	-1	+1	+1	117.5	37.5	451.3	46.3
	15	-1	+1	+1	+1	92.5	52.5	451.3	46.3
	16	+1	+1	+1	+1	117.5	52.5	451.3	46.3
Axial Points	17	-β	0	0	0	80.0	45.0	317.5	42.5
	18	+β	0	0	0	130.0	45.0	317.5	42.5
	19	0	-β	0	0	105.0	30.0	317.5	42.5
	20	0	+β	0	0	105.0	60.0	317.5	42.5
	21	0	0	-β	0	105.0	45.0	50.0	42.5
	22	0	0	+β	0	105.0	45.0	585.0	42.5
	23	0	0	0	-β	105.0	45.0	317.5	35.0
	24	0	0	0	+β	105.0	45.0	317.5	50.0
Centre Points	25	0	0	0	0	105.0	45.0	317.5	42.5
	26	0	0	0	0	105.0	45.0	317.5	42.5
	27	0	0	0	0	105.0	45.0	317.5	42.5
	28	0	0	0	0	105.0	45.0	317.5	42.5
	29	0	0	0	0	105.0	45.0	317.5	42.5
	30	0	0	0	0	105.0	45.0	317.5	42.5

4. Test cyclone, material, rig, procedure, sample and data analyses

The three-product cyclone used for the tests was a 150-mm unit with dimensions of fixed components given in Table 3. The test material was a mixture of magnetite (s.g. = 4.7) and silica (s.g. = 2.7) in which the former comprised approximately 18 % by weight. The 80 % and 20 % passing sizes were 203 and 13 μm respectively. The test rig, procedure and sample analysis were the same as described by Obeng and Morrell (2003). Because the CCRD requires exact positioning of the test points as far as is practicable (Cilliers et al., 1992), the exact levels of the variables given in Table 2 were used. The particle size distributions were measured on the Malvern Mastersizer. The experimental data were mass balanced and used subsequent for analysis.

Table 3 - Dimensions of fixed cyclone components used for the CCRD tests

Cyclone diameter	150 mm
Inlet diameter	36 mm
Diameter of OVF	60 mm
Spigot diameter	25 mm
Cone angle	10^{o}

Dimensions of other cyclone components are given in Table 2.

5. RESULTS AND DISCUSSION

The mass balanced experimental results are summarised in Table A1 in Appendix 1. Results of the regression analysis showing the effect of all the terms (both significant and non-significant) in the response surface model (Equation 1) have been reported (Obeng, 2003). The results in Tables 4-9 show the effect and significance of the individual linear and/or square and/or interaction terms obtained by stepwise refitting the response surface model using only the terms that are significant at greater than or equal to 90 % confidence level, i.e $P(t) \leq 0.1$. Note, however, that in some cases the linear terms, irrespective of their significance, had to be included as the software used required that for every interaction or square term included in the refit, the corresponding linear term must be included. The graphs in Figures 3-8 which are simulations from the response surface model describe the effect of the variables on the performance of the three-product cyclone. The explanations for the trends in Figures 3-8 have been given by Obeng (2003).

5.1 EFFECT OF VARIABLES ON SIZE DISTRIBUTION IN OUO

The influence of the four variables on P80 in the OUO is depicted in Figure 3 while Table 4 gives the effect, along with the significance of the individual terms. The P(t) values in the table show that the effects of the feed percent solids squared and feed percent solids-IVF length interaction terms are significant at 92.4 % (i.e. 1-0.076) and 90 % (i.e. 1-0.100) confidence levels respectively. Note that the linear feed percent solids and LIVF terms are included in this case as a requirement of the software used.

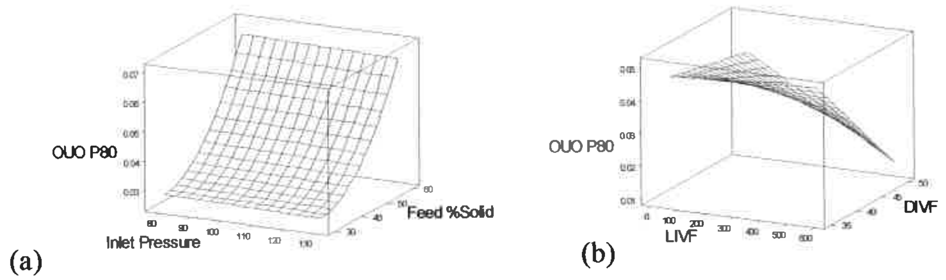

Figure 3. Effect of variables on P80 in the OUO

Table 4: Regression coefficients and ANOVA for P80 in OUO

Term	Coefficient	SE of Coefficient	t	P(t)		
Constant	2.67×10^{-2}	0.048763	0.547	0.589		
Feed % solids	-1.37×10^{-3}	0.001998	-0.683	0.501		
LIVF	1.01×10^{-4}	0.000075	1.355	0.188		
Feed % solids* Feed % solids	4.0×10^{-5}	0.000021	1.848	0.076		
Feed % solids* LIVF	-3.0×10^{-6}	0.000002	-1.686	0.100		
R^2 = 72.5% R^2(Adjusted) = 68.2%						
		ANOVA				
Source	DF	Seq SS	Adj SS	Adj MS	F	P(F)
Regression	4	0.002861	0.002861	0.000715	16.52	0.000
Linear	2	0.002593	0.000127	0.000064	1.47	0.250
Square	1	0.000145	0.000148	0.000148	3.42	0.076
Interaction	1	0.000123	0.000123	0.000123	2.84	0.104
Residual Error	25	0.001083	0.001083	0.000043		
Total	29	0.003943				

Data used for the analysis are given in Table A1 in Appendix 1; the meanings of the acronyms are given in the Nomenclature at the end of this chapter.

Substituting the coefficients in Table 4 in the response surface Equation 1, we obtain the regression equation for P80 in the OUO as:

$$OUOP80 = 2.67\times10^{-2} - 1.37\times10^{-3}\times FS + 1.01\times10^{-4} LIVF + 4\times10^{-5} FS^2 - 3\times10^{-6} FS \times LIVF \qquad (2)$$

where the symbols/acronyms have their meanings given in the nomenclature at the end of this chapter.

The P(F) values from the ANOVA in Table 4 show that the linear, square and interaction terms of Equation 2 are significant at 75 %, 92.4 % and 89.6 % confidence levels respectively, with regression Equation 2 being significant at greater than 99.9 % confidence level.

5.2 Effect of variables on size distribution in INO

Figure 4 shows the influence of the variables on P80 in the INO while Table 5 shows that the effects of the cyclone feed percent solids, LIVF and DIVF are significant at 98.6, 99.9 and 94.2 % confidence levels respectively.

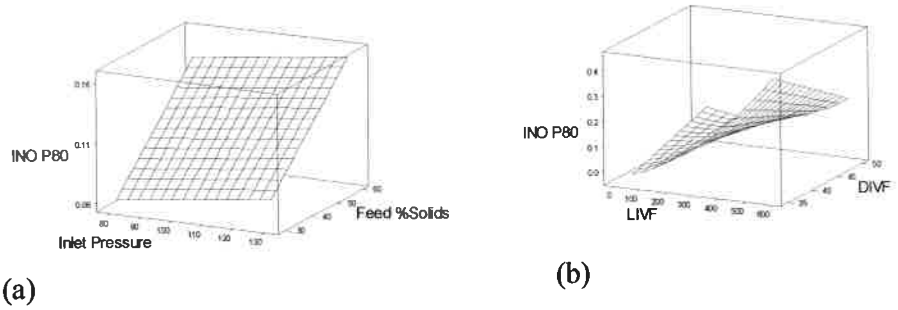

Figure 4. Effect of variables on P80 in INO

Table 5: Regression coefficients and ANOVA for P80 in INO

Term	Coefficient	SE of Coefficient	t	P(t)		
Constant	1.56×10^{-2}	0.103587	0.151	0.882		
Feed % solids	2.8×10^{-2}	0.001057	2.627	0.014		
LIVF	5.1×10^{-4}	0.000059	8.585	0.000		
DIVF	-4.2×10^{-3}	0.002115	-1.979	0.058		
R^2 = 76.5% R^2(Adjusted) = 73.8%						
		ANOVA				
Source	DF	Seq SS	Adj SS	Adj MS	F	P(F)
Regression	3	0.127724	0.127724	0.042575	28.20	0.000
Linear	3	0.127724	0.127724	0.042575	28.20	0.000
Residual Error	23	0.039249	0.039249	0.001510		
Total	29	0.166973				

Data used for the analysis are given in Table A1 in Appendix 1; the meanings of the acronyms are given in the Nomenclature at the end of this chapter.

Substituting the values of the coefficients in Table 5 in Equation 1 gives the regression equation for the P80 in the INO as:

$$INOP80 = 1.56 \times 10^{-2} + 2.78 \times 10^{-3} FS + 5.10 \times 10^{-4} LIVF - 4.20 \times 10^{-3} DIVF \qquad (3)$$

where the symbols/acronyms have their meanings given in the nomenclature at the end of this chapter.

From the ANOVA in Table 5, the linear terms, as well as the regression Equation 3 are significant at greater than 99.9 % confidence level.

5.3 Effect of variables on feed volumetric flowrate

Figure 5 shows the effect of the four variables on cyclone feed volumetric flowrate. The P(t) values in Table 6 show that the effects of the inlet pressure and feed percent solids are significant at greater than 99.9 % confidence level while that of the DIVF is significant at 96.7 % confidence level. The effect of LIVF does not appear in the table as it is not significant in this case.

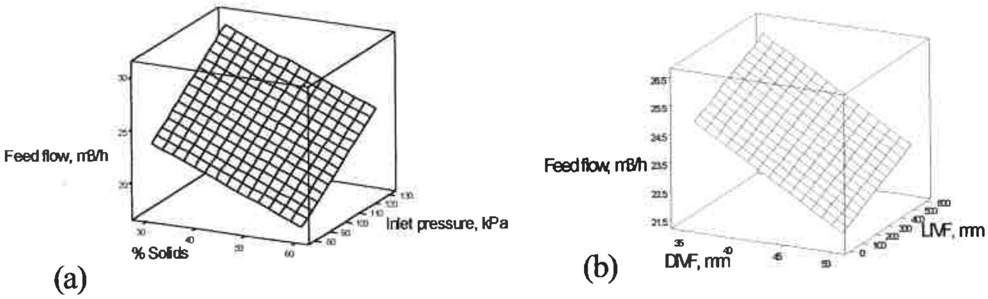

Figure 5. Effect of variables on cyclone feed volumetric flowrate

Table 6: Regression coefficients and ANOVA for feed flowrate

Term	Coefficient	SE of Coefficient	t	P(t)		
Constant	27.23	5.35843	5.081	0.000		
Inlet pressure	0.14	0.02792	4.867	0.000		
Feed % solids	-0.19	0.04652	-4.144	0.000		
DIVF	-0.21	0.09307	-2.252	0.033		
R^2 = 63.9% R^2(Adjusted) = 59.7%						
		ANOVA				
Source	DF	Seq SS	Adj SS	Adj MS	F	P(F)
Regression	3	134.419	134.419	44.8064	15.33	0.000
Linear	3	134.419	134.419	44.8064	15.33	0.000
Residual Error	26	76.010	76.010	2.29235		
Total	29	210.429				

Data used for the analysis are given in Table A1 in Appendix 1; the meanings of the acronyms are given in the Nomenclature at the end of this chapter.

Substituting the coefficients of the terms in Table 6 in the response surface model (Equation 1), we obtain the regression equation for the feed volumetric flowrate (Q_f) as:

$$Q_f = 27.23 + 0.14P - 0.19FS - 0.21DIVF \qquad (4)$$

where the symbols/acronyms have their meanings given in the nomenclature at the end of this chapter.

The P(F) values in Table 6 show that the linear term and the overall regression Equation 4 are significant at greater than 99.9 % confidence level.

5.4 Effect of variables on water recovery to OUO

Figure 6 depicts the influence of the variables on water recovery to the OUO while Table 7 shows that the effects of the cyclone feed percent solids, LIVF, DIVF, DIVF squared and feed percent solids-DIVF interaction terms are significant at 97.7, 99.9, 99.6, 99.9 and 95.5 % confidence levels respectively.

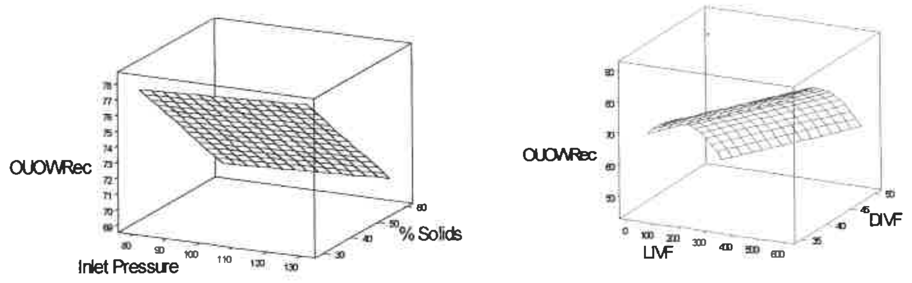

Figure 6. Effect of variables on water recovery to OUO

Substituting the coefficients of the terms in Table 7 in Equation 1, the regression equation for water recovery to the OUO is:

$$OUOW\,\mathrm{Rec} = -22.86 - 1.92 \times FS + 0.03LIVF + 7.91DIVF - 0.13DIVF^2 + 0.04FS \times DIVF \quad (5)$$

where the symbols/acronyms have their meanings given in the nomenclature at the end of this chapter.

The analysis of variance in Table 7 shows that the linear and square terms are both significant at greater than 99.9 % confidence level while the interaction term is significant at 95.5 % confidence level. The regression Equation 5 is also significant at greater than 99.9 % confidence level.

Table 7: Regression coefficients and ANOVA for water recovery to OUO

Term	Coefficient	SE of Coefficient	t	P(t)		
Constant	-22.86	61.2854	-0.373	0.712		
Feed % solids	-1.92	0.7916	-2.428	0.023		
LIVF	0.03	0.0032	9.963	0.000		
DIVF	7.91	2.4934	3.172	0.004		
DIVF * DIVF	-0.13	0.0276	-4.783	0.000		
Feed % solids* DIVF	0.04	0.0189	2.113	0.045		
R^2 = 93.2% R^2(Adjusted) = 91.8%						
		ANOVA				
Source	DF	Seq SS	Adj SS	Adj MS	F	P(F)
Regression	5	1428.14	1428.14	285.628	66.08	0.000
Linear	3	1309.35	532.927	177.642	41.09	0.000
Square	1	99.49	98.884	98.884	22.88	0.000
Interaction	1	19.30	19.300	19.300	4.46	0.045
Residual Error	24	103.75	103.746	4.323		
Total	29	1531.89				

Data used for the analysis are given in Table A1 in Appendix 1; the meanings of the acronyms are given in the Nomenclature at the end of this chapter.

5.5 Effect of variables on water recovery to INO

The influence of the variables on water recovery to the INO is illustrated in Figure 7. Table 8 shows that aside from the effect of the cyclone feed percent solids which is significant at 90 % confidence level, the effects of the LIVF, DIVF and DIVF squared terms are all significant at greater than 99.9 % confidence level.

Substituting the coefficients of the terms in Table 8 in Equation 1, the regression equation for water recovery to the INO is:

$$INOW\,\mathrm{Re}\,c = 228.73 - 0.11FS - 0.03LIVF - 10.99DIVF + 0.15DIVF^2 \qquad (6)$$

where the symbols/acronyms have their meanings given in the nomenclature at the end of this chapter.

The analysis of variance in Table 8 shows that the linear, square terms and the regression Equation 6 are significant at greater than 99.9 % confidence level.

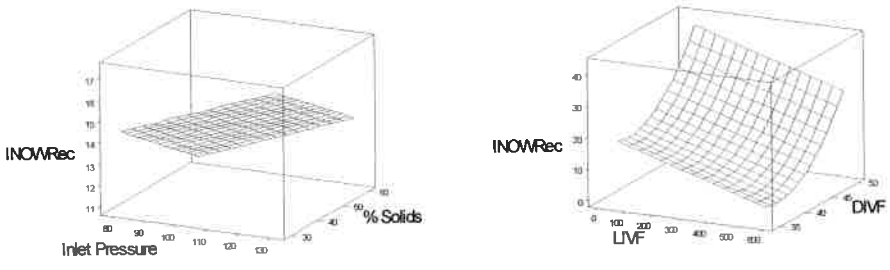

Figure 7. Effect of variables on water recovery to INO

Table 8: Regression coefficients and ANOVA for water recovery to INO

Term	Coefficient	SE of Coefficient	T	P(t)		
Constant	228.73	57.6067	3.971	0.001		
Feed % solids	-0.11	0.0655	-1.637	0.100		
LIVF	-0.03	0.0037	-7.648	0.000		
DIVF	-10.99	2.7135	-4.051	0.000		
DIVF* DIVF	0.15	0.0319	4.623	0.000		
R^2 = 89.8% R^2(Adjusted) = 88.2%						
		ANOVA				
Source	DF	Seq SS	Adj SS	Adj MS	F	P(F)
Regression	4	1277.45	1277.449	319.362	55.15	0.000
Linear	3	1153.68	449.128	149.709	25.85	0.000
Square	1	123.77	123.765	123.765	21.37	0.000
Residual Error	25	144.76	144.763	5.791		
Total	29	1422.21				

Data used for the analysis are given in Table A1 in Appendix 1; the meanings of the acronyms are given in the Nomenclature at the end of this chapter.

5.6 Effect of variables on water recovery to underflow

The effect of the variables on water recovery to the underflow is depicted in Figure 8 with Table 9 showing that the effects of the LIVF and cyclone feed percent solids-LIVF interaction terms both being significant at greater than 99.4 % confidence level. Note that the cyclone feed percent solids term appears in this case as a requirement of the software used.

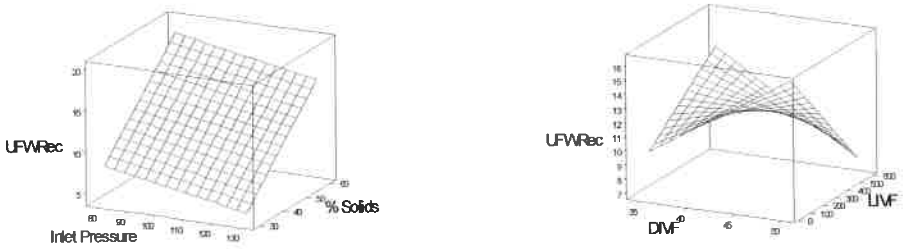

Figure 8. Effect of variables on water recovery to recovery to underflow

Table 9: Regression coefficients and ANOVA for water recovery to underflow

Term	Coefficient	SE of Coefficient	t	P(t)		
Constant	20.2661	8.25823	2.454	0.021		
Feed % solids	-0.1630	0.18253	-0.893	0.380		
LIVF	-0.0780	0.02475	-3.150	0.004		
Feed % solids* LIVF	0.0017	0.00055	3.036	0.005		
R^2 = 64.4% R^2(Adjusted) = 60.3%						
ANOVA						
Source	DF	Seq SS	Adj SS	Adj MS	F	P(F)
Regression	3	225.084	225.084	75.028	15.71	0.000
Linear	2	181.062	212.493	106.246	22.24	0.000
Interaction	1	44.022	44.022	44.022	9.21	0.005
Residual Error	26	124.211	124.211	4.777		
Total	29	349.295				

Data used for the analysis are given in Table A1 in Appendix 1; the meanings of the acronyms are given in the Nomenclature at the end of this chapter.

Substituting the coefficients of the terms in Table 9 in Equation 1, the governing regression equation for water recovery to the underflow is:

$$UFW\,\mathrm{Re}\,c = 20.27 - 1.63\times10^{-1} FS - 7.80\times10^{-2} LIVF + 1.70\times10^{-3}\times FS\times LIVF \quad (7)$$

where the symbols/acronyms have their meanings given in the nomenclature at the end of this chapter.

The analysis of variance in Table 9 shows that the linear and interaction terms are significant at 99.9 % and 99.5 % confidence levels respectively, the regression Equation 7 being significant at greater than 99.9 % confidence level.

6. CORRELATION BETWEEN PREDICTED AND OBSERVED RESPONSES

Figure 9 shows the correlation between the model-predicted and observed responses for Equations 2 to 7. As can be seen from the figure and the R^2 values in Tables 4-9, the equations fit the experimental data reasonably well. The analyses of variance (ANOVA) in Tables 4-9 also show that the linear, square and/or interaction terms, as the case may be, are generally significant, the resultant regression equation in each case being significant at greater than 99.9 % confidence level, i.e. $P(F) \approx 0.00$.

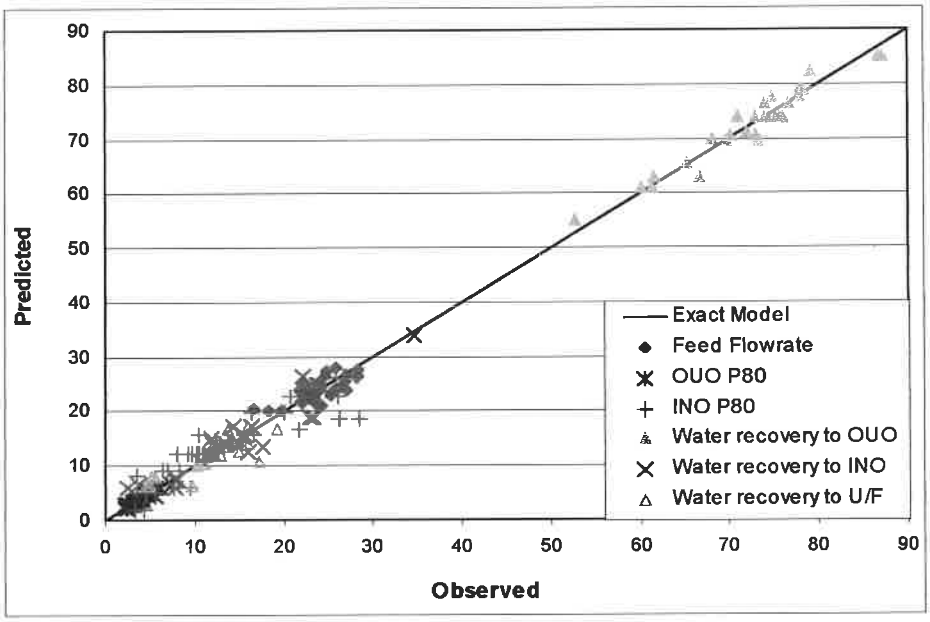

Figure 9. Correlation between the predicted and observed responses

The above attributes show that the influence of the variables as described by the regression Equations 2-7 is significant. Hence they can be used to estimate the response for variations in IVF length and diameter, cyclone inlet pressure and feed percent solids not included in the experimental design but fall within the range of conditions given in Section 3.

It is worth noting that the trends obtained for the influence of feed percent solids on water recovery to the product streams and IVF length on size distribution in the overflows are consistent with those obtained from tests with other feed materials (Obeng, 2003; Obeng and Morrell, 2003).

7. CONCLUSIONS

The Central Composite Rotatable Design (CCRD) has been successfully applied to the design of an experimental program to establish the general process trends, determine the statistical significance and model the effect of IVF length and diameter, inlet pressure and feed percent solids on the performance of the three-product cyclone. The number of experiments required for the CCRD was 51 less than that required for a three-level full factorial design.

The effects of the length and diameter of the second vortex finder, and the cyclone feed percent solids were found to be significant in most cases. The resultant model is also significant, predicts the experimental data well and can be used to estimate the response corresponding to operating conditions not included, but fall within the range of conditions in the experimental design.

The trends obtained for the influence of IVF length on size distribution in the overflows and feed percent solids on water recovery to the product streams were consistent with those obtained from tests with other feed materials.

NOMENCLATURE

DIVF:	Cyclone inner vortex finder diameter (m)
INO	Inner overflow stream
IVF	Inner vortex finder
LIVF:	Length of inner vortex finder (m)
F_{20}:	feed 20 % passing size (mm)
F_{80}:	feed 80 % passing size (mm)
FS:	feed percent solids
OUO:	Outer overflow stream
OVF:	Outer vortex finder
P:	Cyclone inlet pressure (kPa)
Q_f:	feed flowrate (m^3/h)
R_f:	Water recovery to underflow (%)

ACRONYMS FOR TABLES 4-9:

Adj MS:	Adjusted mean square
Adj SS:	Adjusted sum of squares
ANOVA:	Analysis of variance
DF:	Degrees of freedom
F:	F- statistic
P(F):	Probability value of F- statistic indicating level of significance
P(t):	Probability value of t- statistic indicating level of significance
R^2:	Coefficient of determination

R^2(Adjusted): Coefficient of determination adjusted for degrees of freedom

SE of Coefficient: Standard error of coefficient

Seq SS: Sequential sum of squares

t: t- statistic

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APPENDIX 1

Table A1 - Mass balanced experimental response data

Test	Feed flowrate, m^3/h	P80OUO, mm	P80INO, mm	OUOWRec, %	INOWRec, %	UFWRec, %
1	24.1	0.031	0.035	76.7	11.8	11.5
2	25.8	0.033	0.046	74.0	15.7	10.4
3	23.4	0.056	0.063	73.1	12.4	14.5
4	26.4	0.042	0.285	86.7	7.8	5.5
5	22.2	0.031	0.033	66.6	22.2	11.2
6	22.2	0.059	0.083	70.2	17.7	12.1
7	28.2	0.042	0.263	87.2	8.0	4.8
8	24.6	0.026	0.043	61.5	27.4	11.1
9	22.0	0.049	0.26	78.3	2.4	19.4
10	18.2	0.049	0.075	60.1	24.0	15.9
11	25.2	0.024	0.155	71.9	22.9	5.2
12	23.7	0.047	0.207	78.0	7.8	14.2
13	22.8	0.047	0.095	61.3	23.8	14.9
14	28.3	0.022	0.166	72.1	23.4	4.5
15	19.7	0.037	0.165	69.8	16.5	13.8
16	23.3	0.041	0.201	73.4	14.3	12.3
17	16.5	0.038	0.079	71.0	15.2	13.8
18	28.1	0.035	0.109	75.1	14.5	10.4
19	27.0	0.022	0.034	74.8	15.6	9.6
20	24.3	0.073	0.218	68.1	15.9	16.0
21	23.9	0.055	0.056	65.2	22.8	11.9
22	21.7	0.031	0.259	79.0	3.7	17.3
23	26.8	0.046	0.104	77.9	10.7	11.4
24	23.3	0.025	0.07	52.6	34.6	12.8
25	24.1	0.036	0.104	75.6	13.1	11.3
26	24.8	0.035	0.102	72.9	14.9	12.2
27	24.1	0.04	0.11	74.4	13.5	12.1
28	24.9	0.04	0.104	76.2	12.1	11.7
29	24.6	0.037	0.097	74.0	14.6	11.4
30	24.5	0.037	0.093	72.9	15.2	11.9