How to Formulate an Effective Ore Comminution Characterization Program

S. Morrell
Managing Director, SMC Testing Pty Ltd, Brisbane, Australia, steve@smccx.com

ABSTRACT

Whether it is for the purposes of the design of new comminution circuit, a brownfield expansion or a geometallurgical program, one of the principal keys to success is to “know your orebody”. To do so requires the formulation and execution of a drill core sampling campaign followed by a laboratory program where ore samples are subjected to appropriate comminution tests. When contemplating such programs three of the most often-asked questions are: “What laboratory comminution test(s) should be carried out?”, “How many samples will be required?” and “Where should the samples be taken from”. This paper provides guidance to help answer these questions and in the course of doing so reviews some of the more popular comminution tests currently available, how effective they are and the extent to which comminution characteristics may vary throughout orebodies. The degree to which comminution characteristics vary is illustrated using statistics from a data base of 50,000 test results covering over 1,800 orebodies. These results are used to provide guidelines to help decide on the requisite number of samples required to ensure that an orebody is appropriately characterized from a comminution perspective.

INTRODUCTION

The last 20 years has seen an increasing awareness of the need for detailed ore characterization data with which to map orebody hardness, regardless of the nature of the project. In the case of new project development the need to understand the breakage characteristics of the orebody is paramount if the comminution circuit is to be sized correctly for the duration of the life-of-mine. The need is no less different in geometallurgy projects where it is required to forecast the performance of comminution circuits. Regardless of the end use when planning the undertaking of an ore comminution characterisation programme the three most common questions that are initially considered are:

1. What test(s) should be used?
2. How many samples should be tested?
3. Where should the samples be taken from within the orebody?

In the following sections each of these questions will be considered in turn.

WHAT TEST(S)?

Before this question can be answered it should be first determined what final use the test results will be put to. Regardless of whether the characterization programme is for a greenfield design, a brownfield expansion or a geometallurgical model, the end use of the characterization test result will involve it being applied in an equation/model that predicts the comminution machine/circuit specific energy. To maximise the chances of success, of necessity the chosen equation/model must be able to convincingly demonstrate it has a high degree of accuracy. This can only be done through validation using an extensive data base of real plant data. It is outside the scope of this paper to describe the equations/models that are associated with the various commercially available characterization tests and what their accuracies are. However, the reader is directed to the SME Mineral Processing and Extractive Metallurgy Handbook (SME, 2019), which contains a detailed review of this subject.

An associated issue that must also be addressed early in the planning stages is to ensure that, whatever equations/models and associated characterisation tests are chosen, they are fit for purpose in that they...
relate to the equipment/circuit in question. This may be seen as stating the obvious but it is the author’s experience that in some cases this issue has not been fully thought through and a characterisation programme has been executed in which, say, a number of Bond rod and ball mill work index tests have been carried out without considering that the design study was to include a trade-off study between an SABC and a Crushing/HPGR/ball mill circuit. Hence in this particular example, rather than rod mill tests being carried out, tests suitable for crushers, AG/SAG mills and HPGRs should have been substituted. Table 1 provides guidance for such situations and shows the most popular comminution tests commercially available and what comminution equipment they can be used for.

Table 1 – Most Popular Commercially Available Laboratory Comminution Tests

<table>
<thead>
<tr>
<th>Test</th>
<th>Use</th>
<th>References</th>
<th>Sample mass</th>
<th>Material Size</th>
<th>Min Core</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond crushing</td>
<td>Conventional crushers</td>
<td>Bond (1961)</td>
<td>40 kg</td>
<td>-75+50mm</td>
<td>HQ</td>
</tr>
<tr>
<td>Bond rod mill</td>
<td>Rod mills / ball mills</td>
<td>Bond (1961)</td>
<td>15 kg</td>
<td>100% -12.7 mm</td>
<td>NQ</td>
</tr>
<tr>
<td>Bond ball mill</td>
<td>Ball mills</td>
<td>Bond (1961)</td>
<td>10 kg</td>
<td>100% -3.35 mm</td>
<td>NQ</td>
</tr>
<tr>
<td>SPI®</td>
<td>SAG mills</td>
<td>Dobby et al. (2001)</td>
<td>12 kg</td>
<td>100% -19 mm</td>
<td>NQ</td>
</tr>
<tr>
<td>JK Drop-weight</td>
<td>AG/SAG mills</td>
<td>Napier-Munn, et al. (1996)</td>
<td>100 kg</td>
<td>-63+13.2 mm</td>
<td>HQ</td>
</tr>
<tr>
<td>SMC Test*</td>
<td>AG/SAG mills</td>
<td>Morrell (2004a)</td>
<td>5-20 kg</td>
<td>-31.5+13.2 mm</td>
<td>NQ</td>
</tr>
</tbody>
</table>

Bond Crushing Work Index Test

For crushing applications Bond developed an apparatus comprising two opposing 30-lb hammers that came together through the action of two counter-rotating wheels (see Figure 1) (Bond, 1961). The wheels were 22-in. front bicycle wheels, reinforced with steel bands around each rim. The left-hand wheel was initially rotated clockwise and, through a connecting device, the right-hand wheel was synchronized and rotated counterclockwise by a similar amount. As the hammers were attached to the wheels, the hammers were lifted up by this rotation, and when the wheels were released, the hammers would collide with one another at the 6 o’clock position. By measuring how high the hammers were raised in relation to their final rest position, the potential energy could be estimated. A rock specimen was mounted on a plinth at the point of collision of the hammers and hence was broken with an amount of energy equivalent to the estimated potential energy. Figure 1 is a copy of a picture of the device from a very old source and hence is not of ideal quality. Where the hammers collide (center of the V-frame toward the bottom of the picture) is therefore not clear. Figure 2 shows a clearer view of this region and has been taken from a modern machine that is faithful to Bond’s original design.
Bond specified that only particles in the size range of –3 in. + 2 in. should be tested in this machine. Particles outside of this range should not be used. Bond also stated that slabby or acicular pieces should not be used and recommended that a minimum of 10 suitable rocks should be broken. For breaking each specimen Bond quoted the following procedure: “The first piece is tested with a low-energy blow and the height of fall is gradually increased until the specimen breaks into two or more pieces of approximately equal size. Each succeeding piece is first tested with an energy slightly under that required to break the preceding piece and the height of fall is increased so that the specimen is broken after two or three blows.” The aim of the test is therefore to try to measure the energy needed to just break the rock.

Bond’s crushing work index is calculated using equation 1, based on the average result from the 10 rocks.

\[
W_{ic} = 2.85C/sg
\]  

(1)

where

- \( W_{ic} \) = Bond crushing work index (kWh/t)
- \( C = E/D \)
- \( E \) = breakage energy (ft-lb)
- \( D \) = rock thickness (in.)
- \( sg \) = specific gravity

The \( W_{ic} \) value obtained from equation 1 is used with Bond’s general power-based size reduction equation (eq. 2) which predicts the specific energy of conventional crushers (Bond, 1961).

\[
W = 10Wi (P^{-0.5} - F^{-0.5})
\]  

(2)

where

- \( W \) = specific motor output energy (kWh/t)
- \( Wi \) = work index (kWh/t) as determined from the relevant laboratory test
- \( P \) = 80% passing size of the product in \( \mu \)m
- \( F \) = 80% passing size of the feed in \( \mu \)m
The Bond crushing work index has very poor precision which may result in extremely large differences between labs treating the same material (Angove and Dunne, 1997). Angove and Dunne reported that for some samples the coefficient of variation (standard deviation/mean) was as high as 43.5%, being a reflection of results from some laboratories being well over double those from another laboratory treating the same material. These results are supported by further test results from another programme that the author was involved in during which over 20 different samples were sent to one laboratory and the same number of duplicate samples were sent to another. The results from one of the laboratories were on average over 75% higher than from the other, with individual result differences being up to 280%. The reasons for this very poor precision are twofold, the first being due to the nature of the test, which is very operator sensitive. The second reason is the lack of test equipment standardization, there being large differences in test machine design between some laboratories (Bailey et al, 2009).

**Bond Rod Milling Test**

To characterize rock breakage in rod mills, Bond developed a dry locked-cycle test that uses a 12-in.-diameter, 24-in.-long batch mill with wave liners and running at 46 rpm (see Figure 3). The mill is mounted on a rocker, which allows for the mill to be tilted up by 5° to the horizontal and down by 5° during the course of the test. The mill is charged with a specified quantity of rods of given sizes. The rock being characterized is stage-crushed to 100% passing 12.7 mm and dry-ground in closed circuit with a screen of aperture $P_1$ such that the recycle load is 100%. The net grams of final product produced per revolution when the recycle load stabilizes at 100% is measured and inserted into equation 3 to obtain the rod mill work index.

\[
W_r = \frac{68}{P_1^{0.22} (G_{rp})^{0.25} 10 (\frac{1}{\sqrt{P}} - \frac{1}{\sqrt{F}})}
\]

where

- $W_r =$ rod mill work index (kWh/t)
- $P_1 =$ closing screen size in µm
- $G_{rp} =$ net grams of screen undersize per mill revolution
- $P =$ 80% passing size of the product in µm
- $F =$ 80% passing size of the feed in µm
The Wir value obtained from equation 3 is used with Bond’s general power-based size reduction equation (eq. 2) which, together with some additional factors (Rowland, 1982) predicts the specific energy of rod mills.

According to Angove and Dunne (1997) the precision of the rod mill work index test is not as poor as that of the crushing work index test. However they did report coefficients of variation as high as 11.4%. This is supported by further test results from another programme which the author was involved in during which over 20 different samples were sent to one laboratory and the same number of duplicate samples were sent to another. The results from one of the laboratories were on average over 30% higher than from the other, with individual result differences being up to 65%. The reasons for this relatively poor precision are believed to be due to differences in test mill design and operating condition, such as those associated with wave-liner number and profile, size of rods and whether the mill is rocked or not during operation. Bond unfortunately never specified the profile and number of wave liners in his original mill and so laboratories have no definitive guidance in this matter. In some cases this has led laboratories to not use a wave profile at all but to use smooth liners.

![Figure 3 - Bond rod mill](image)

**Bond Ball Milling Test**

For ball milling, Bond (1961) also developed a dry locked-cycle test similar to that for rod milling. In this case he used a 12-in.-diameter, 12-in.-long batch mill with rounded corners and smooth liners, running at 70 rpm (see Figure 4). The mill is charged with a specified quantity of balls of given sizes. The rock being characterized is stage-crushed to 100% passing 3.35 mm and ground in closed circuit with a screen of aperture $P_1$ such that the recycle load is 250%. The net grams of final product produced per revolution when the recycle load is stable at 250% is measured and inserted into equation 4 to obtain the ball mill work index. Bond recommended that the screen aperture ($P_1$) chosen to close the test with should be such that it gives a final product $P_{80}$ similar to that being targeted in the full-scale plant. Typically, the final product $P_{80}$ is of the order of $1/\sqrt{2}$ of the closing screen aperture.
\[ W_{ib} = \frac{49}{P_1^{0.23}(Gbp)^{0.82}} \left( \frac{1}{\sqrt{P}} - \frac{1}{\sqrt{F}} \right) \]  \hspace{1cm} (4)

where

- \( W_{ib} \) = Bond laboratory ball work index (kWh/t)
- \( P_1 \) = closing screen size in \( \mu m \)
- \( Gbp \) = net grams of screen undersize per mill revolution
- \( P \) = 80% passing size of the product in \( \mu m \)
- \( F \) = 80% passing size of the feed in \( \mu m \)

The \( W_{ib} \) value obtained from equation 4 is used with Bond’s general power-based size reduction equation (eq. 2) which, together with some additional factors (Rowland, 1982) predicts the specific energy of ball mills.

Morrell, in his approach in predicting ball mill specific energy (Morrell, 2004b) also uses the raw data from Bond’s laboratory ball mill test but uses equation 5 to obtain a “Morrell” work index (\( M_{ib} \)). This value is used in equation 7 (see later) to predict the specific energy of ball mills.

\[ M_{ib} = \frac{18.18}{P_1^{0.295}(Gbp) \left( P_{80} \frac{f(P_{ib})}{P_{ib}} - F_{80} \frac{f(F_{ib})}{F_{ib}} \right)} \]  \hspace{1cm} (5)

where

- \( M_{ib} \) = Morrell work index for grinding of finer particles (<750\( \mu m \)) (kWh/t)
- \( P_1 \) = closing screen size in \( \mu m \)
- \( Gbp \) = net grams of screen undersize per mill revolution
- \( P_{80} \) = 80% passing size of the product in \( \mu m \)
- \( F_{80} \) = 80% passing size of the feed in \( \mu m \)
The ball mill work index test has the best precision of all of the Bond laboratory tests. The most detailed study of precision that has been published was carried out by Weier and Chenje (2018). In their study they carefully prepared samples of the same material and sent them to 38 different laboratories. Their results showed a coefficient of variation of 7.1%. This is supported by further test results from another programme during which over 20 different samples were sent to one laboratory and the same number of duplicate samples were sent to another. The results from one of the laboratories were on average over 13% higher than from the other.

**SAG Power Index (SPI®)**

The SAG power index (SPI) was originally developed in the 1990s (Starkey et al. 1994). The test uses a 12 in. × 4 in. (D × L) batch laboratory mill, loaded with 15% by volume of 1-in. balls. A picture of the mill is shown in Figure 5. The mill is loaded with 2 kg of –19 mm (P80 = 12.7 mm) of dry sample and run until it is ground to 80% passing 1.7 mm. The time in minutes taken to reach this grind size is designated the SPI value. An additional laboratory crushing test also must be performed, which requires a further 10 kg of material and generates a crushing parameter (Ci), the details of which are proprietary. The SPI and Ci are used in proprietary equations for predicting the specific energy of SAG mills (Dobby et al, 2001).

![Figure 5 - SPI Laboratory Test Mill (After Amelunxen, 2003)](image)

**JK Drop-Weight Test**

The JK drop-weight test (JKDWT) uses the JK drop weight tester, which was originally developed in 1992 (Napier-Munn et al. 1996) and was driven by the need for a machine that could break relatively large rocks, was simple to use, was easy to maintain, and was relatively precise (i.e., had good repeatability). The device (Figure 6) comprises an impact head with a hardened steel face, which can be raised to a range of heights up to approximately 1 m. The mass of the impact head can also be varied. The impact head is raised and a single rock is placed on a hardened steel anvil directly under the impact head. The impact head is then released and falls under the action of gravity and impacts and breaks the target particle (Figure 7). Through a combination of different impact head masses and heights a very wide range of energies can be generated with which to break rocks.

The test typically requires about 100 kg of sample and involves breaking rocks from five different size
fractions: –63+53 mm, –45+37.5 mm, –31.5+26.5 mm, –22.4+19 mm, –16+13.2 mm. If the test is conducted on drill core, it normally requires PQ (85 mm) core to provide sufficient material for the largest size fraction, though with care whole HQ core may sometimes suffice. Each rock size fraction is broken with a range of three input energies and the resultant broken particles are sized. In total, 15 sets of data are generated per test (5 size fractions × 3 energy levels).
The purpose of the test is to generate relationships between the specific energy used to break rocks and the size distribution of progeny rocks. This is done using a the $t_{10}$ parameter which is related to both the specific energy and the size distribution of the progeny rocks. The relationship between $t_{10}$ and specific energy is given in equation 6, whilst the relationship between the $t_{10}$ and the progeny size distribution is shown in Figure 8.

$$t_{10} = A \left( 1 - e^{-b.Ecs} \right)$$

where
- $t_{10} = \%$ passing 1/10th of the original particle’s diameter
- $A$ and $b =$ ore-specific parameters which are fitted to the drop-weight test raw data
- $Ecs =$ specific comminution energy (kWh/t)

![Figure 8 – Relationship Between the $t_{10}$ and the Progeny Size Distribution](image)

Equation 6 and the relationship in Figure 8 are used in simulation models of AG/SAG mills and crushers using the comminution simulator JKSimMet. The product $A \times b$ of the ore-specific parameters are also used in a comparative manner to indicate ore hardness, lower values indicating harder ores.

The JKDWT is a high-precision test as evidenced by the results from extensive “round robin” programmes executed by JKTech. The results from the latest programme (Matei et al, 2015) which involved 28 different laboratories show that the coefficient of variation is 3.82%.

Given the relatively large sample requirement and the relatively large rock sizes also required, the test is not suitable for use with small-diameter drill core. In cases where only small diameter drill core is available and JKSimMet modeling parameters are required, the SMC Test® can be used.

**SMC Test®**

The SMC Test was developed by Morrell (2004a) to provide a range of comminution parameters from the breakage of relatively small amounts of small-diameter drill core. SMC is an acronym for Steve Morrell Comminution (not SAG Mill Comminution as is incorrectly stated in some published literature).
Normally 5–20 kg of drill core (or rocks) are required by the laboratory to conduct the test depending on the source of material. From this 1–5 kg is extracted for the actual test depending on the size fraction chosen to do the test with. The broken material from the test can be added back to the original sample and reused to conduct a standard Bond ball work index test.

The test uses the JK drop weight testing device to break a suite of 100 closely sized particles at five different energy levels. The particle size used for the test is chosen from one of the following three fractions: −31.5 + 26.5 mm, −22.4 + 19 mm, or −16 + 13.2 mm. The choice of which size fraction to use depends on the source material; for example, if only quartered (slivered) BQ (38 mm) core is available, then the smallest size fraction would be chosen, while if larger diameter core were available then the coarser size fractions would be used. In general, where sample size/quantity permits, the largest size fraction should be used. The particles for the test can be obtained using two different feed preparation routes. If a relatively small quantity of drill core sample is available, then the particles can be produced by cutting the drill core into wedges (Figure 9). If sample quantity is not a problem, then the particles are produced by “light” stage-crushing of the core and selecting the required sizes via sieving the product at the end of each stage (Figure 10). Research has shown that there is no difference in results from using either diamond-cut or crushed material (see www.smctesting.com/about/technical-information).
The raw data from the SMC Test such as shown in Figure 11 are processed, and from the results a range of comminution parameters are generated. These parameters fall into two groups. The first group contains parameters that are used in power-based equations for predicting the specific energy of comminution machines. This group includes the following comminution indices:

- **DWi** - a strength index related to the point load index and UCS
- **Mic** - a specific energy index related to size reduction in convention crushers
- **Mia** - a specific energy index related to size reduction of coarser particles (>750 µm) in primary mills such as AG and SAG mills, rod mills and primary ball mills
- **Mih** - a specific energy index related to size reduction in high pressure grinding rolls

The Mi indices are used with the following power-based equation which takes the general form as shown below. Depending on the comminution equipment and circuit being considered various additional factors may be applied (Morrell, 2009, 2010).

\[
W_i = M_i \cdot 4 \left( \frac{x_2^{f(x_2)}}{x_1^{f(x_1)}} - \frac{1}{x_1^{f(x_1)}} \right)
\]  

(7)

where

- \(W_i\) = specific comminution energy (kWh/t)
- \(M_i\) = “Morrell” specific energy indices: Mia, Mib, Mic, Mih
- \(x_2\) = 80% passing size for the product in µm
- \(x_1\) = 80% passing size for the feed in µm
- \(f(x_j) = -0.295 + x_j/1,000,000\); \(j=1,2\)

The second group of parameters that the SMC Test generates are used for simulation modeling purposes, notably for the AG/SAG mill and crusher models contained in the comminution simulator JKSimMet. This group includes the JK rock breakage parameters A and b as well as the JK crusher model’s t10-specific energy matrix.

As with the JKDWT, the SMC Test is also a high-precision test as evidenced by the results from extensive “round robin” programmes executed by JKTech. The results from the latest programme (Matei et al, 2015) which involved 28 different laboratories show that the coefficient of variation is 4.88%, only 1% more than that from the JKDWT.
SAMPLE SELECTION

How Many Tests?

There is no simple answer to this question as it depends on factors such as the accuracy and resolution required from predictions made using the hardness results, the inherent hardness variability of the ore body and how the hardness is distributed throughout the ore body. The accuracy is particularly important and in a greenfield design situation will depend upon what stage the design study is at. Most mining companies will have at least 3 study stages with each successive stage having an increasingly stringent accuracy requirement. Mining companies have differing accuracy requirements for each stage and also differing names for them. Table 2 gives details of the more common naming conventions used for each stage together with the associated typical accuracy ranges. A 3-stage schedule is shown though some companies may have 4 stages, with the feasibility stage divided into an initial “Feasibility” stage followed by a “Definitive” (DFS) or “Bankable” (BFS) feasibility stage. In such cases the Scoping and Pre-feasibility stage accuracy would tend to require the less stringent values shown in Table 1 (say 50% and 35% respectively), whilst the Feasibility stage would require 20-25% accuracy and the DFS/BFS 10-15%.

Table 2 – Typical Accuracies Required in Design Studies

<table>
<thead>
<tr>
<th>Stage</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scoping/Conceptual/Identification</td>
<td>+/- 30-50%</td>
</tr>
<tr>
<td>Pre-feasibility/Selection</td>
<td>+/- 20-35%</td>
</tr>
<tr>
<td>Feasibility/Definition</td>
<td>+/- 10-15%</td>
</tr>
</tbody>
</table>

The accuracies in Table 2 can be related directly to the number of samples required through the use of simple (classical) statistics as shown in equation 8.

Accuracy (%) = \( t_{c(l-1)} \times \frac{c_{v_s}}{\sqrt{n}} \)  \( (8) \)

Where

- \( t \) = value of the \( t \) statistic at confidence level, \( c_l \), and \( n-1 \) degrees of freedom
- \( n \) = number of rock samples tested
- \( c_{v_s} \) = coefficient of variation (%) determined from the \( n \) rock samples tested
In a well designed and thought-through programme the results from the tests on rock samples taken at the scoping stage should be carried through to the pre-feasibility stage and so-on to the final feasibility stage. By way of illustration as to how this can be done, the following example is considered.

**Worked Example:**

A comminution circuit design study for the treatment of a copper ore is mounted. The study has three stages with required accuracies as given below:

<table>
<thead>
<tr>
<th>Stage</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scoping</td>
<td>+/-30%</td>
</tr>
<tr>
<td>Pre-feasibility</td>
<td>+/-20%</td>
</tr>
<tr>
<td>Feasibility</td>
<td>+/-10%</td>
</tr>
</tbody>
</table>

The number of hardness characterization samples for each stage is required to be determined so that a comminution circuit can be selected. Initially nothing at all is known about what the mean hardness of the deposit is nor what the variation about the mean is. The job of the scoping study is to start the process of accumulating such information. With reference to equation 8 the target accuracy for this stage is stipulated as +/-30% but as nothing is yet known about the deposit variability, the coefficient of variation is unknown. An initial guess of what this value might be must hence be made. For this purpose Figure 12 is used. This figure shows the histogram representing the distribution of coefficients of variation from the 1800+ deposits in the SMC Test data base. It is bi-modal with a mean of approximately 31% and for this exercise it is assumed that our copper deposit has a coefficient of variation of 31%. Using this value in equation 8 and a confidence level (2-tailed) of 95%, the indicated number of rock samples is 7. Data from 7 samples were then selected at random from a copper deposit in the SMC Test data base which had been thoroughly characterized (well over 2000 rock samples tested). The mean and sample standard deviation were calculated from these 7 data points and the resulting coefficient of variation was 46.5%. This value was then carried through to the next stage of the project (pre-feasibility) as the best estimate of the deposit variability. The target accuracy for the pre-feasibility stage is +/-20% and equation 8 returns a total of 23 rock samples being required to obtain this accuracy. As 7 samples have already been taken, this stage therefore needs an additional 16. 16 additional data points were then selected at random from the SMC Test deposit which, when combined with the previous 7 gave a coefficient of variation of 34.1%. Carrying these results forward to the final feasibility stage with a target accuracy of +/-10%, yields a value of 47 rock samples being required in total. This equates to an additional 24 rock samples. Randomly selecting a further 24 data points from the SMC Test copper deposit and combining them with the previous 23 rock samples provided an estimated coefficient of variation of 36.6%. This is our best estimate of the coefficient of variation for the deposit and is slightly higher than our estimate of 34.1% going in to this final stage of the project. We therefore need to go back to equation 8 with this revised value and determine what effect this has had on expected accuracy for this final stage. Equation 8 indicates that the expected accuracy is 11%. Depending on the risk profile of the company we can either accept this or increase the total number of required rock samples to 54 (ie an additional 7), as equation 8 predicts that this is the required number to obtain 10% accuracy when the estimated coefficient of variation is 36.6%. If we do this then the final results from our sampling campaign indicate a deposit coefficient of variation of 34.8%. Out of interest the mean DWi of the 54 rock samples was 6.25 kWh/cu.m compared to the true mean value (from the 2000+ samples in the SMC Test copper deposit) of 6.48 kWh/t and a true coefficient of variation of 34.2%. 
Equation 8 and the worked example are based on the application of classical statistics which rely on a number of underlying assumptions to be true. One of these is that the property in question (in this case hardness) is randomly distributed throughout the deposit. For some deposits this is quite clearly not true. For example some open-pit deposits have a relatively clearly defined weathered upper oxide layer which is much harder than lower unweather sulphides. Instead of this relatively simple type of horizontal boundary other deposits can have more complex vertical and horizontal boundaries such as shown in Figure 13. These may define a number of domains which have significantly different hardnesses to one another. Other deposits, rather than having clearly defined boundaries may exhibit a gradual and systematic variation in hardness with depth such as shown in Figure 14. Despite this added layer of complexity it is still possible to apply equation 8 as it may well be that within hardness domains hardness values are randomly distributed. One of the initial problems is therefore to identify such domains and sample them independently. Geology may help indicate where such domains might exist so that in the earlier stages of the project samples can be taken which specifically target these zones. For example, say four main ore types are identified from a geological perspective then part of the pre-feasibility programme might be to initially carry out scoping-study levels of sampling of each ore type. If the results indicate significant differences in ore hardness between ore types then the pre-feasibility study may need to progress by looking at each ore type independently. This will inevitably lead to an increase in the number of rock samples that are required.

In some cases the existence of “proxy” data can help limit the number of samples required by helping to identify hardness domains early in the study. Such proxies include hardness data collected for other purposes such as point load indices (IS50) and UCS values which mining and geotechnical engineers collect for blasting and wall stability purposes. Figure 15 shows a deposit where a correlation was established between the IS50 and the DWi which could be used to help identify hardness domains. In some deposits sg is correlated with hardness and can be used for a similar purpose (Figure 16), whilst in others chemical and/or mineralogical relationships can be found.
The number of rock samples required also depends on the purpose that the tests will be put to, in particular the resolution required. For instance in the worked example given earlier in the paper the programme would allow us to make conclusions/do calculations “on average” concerning the overall volume of the ore body the samples were taken from. It would not, for example, allow us with +/- 10% accuracy and 95% confidence to say what the ore hardness would be for the first year of plant operation. To do that all of the samples would have had to have been taken from the volume of the ore body that was to be processed in the first year. To compensate for this some companies will bias their sampling programme so that there is a relatively high proportion of tests in ore that will be processed in the early years (typically three or so years) so that they can be more confident that in this extremely important initial period of operation the chosen comminution circuit will be able to process the ore at the desired rate. Once the plant is in operation a geometallurgical model can then be constructed and an on-going ore characterization programme started that targets ore that will be treated in future years. Such geometallurgical models may require to forecast production with a month-by-month resolution and hence the number of samples required will be significantly greater than the requirement for initial design purposes. As a result those companies that take ore characterisation very seriously and have developed successful geometallurgical programmes conduct large numbers of ore characterization tests on a routine yearly basis. Examination of the SMC Testing data base indicates that generally speaking South American copper and gold producers are world leaders in this area. As a very rough guide the larger companies of this region routinely test of the order of 1 sample for every 200,000 tonnes of ore. Hence in the case of, say, a 1000 tph comminution plant running for a nominal 8000 hours per year this equates to 40 tests per year and for a 5000 tph plant 200 tests per year would be conducted. If we assign a total cost of $US2000 to characterise a sample then this equates to a cost of only 1c per tonne of ore milled.

![Figure 13 – Geology of the Antamina Deposit (after Dechert, 2006)](Image)
Figure 14 – Example of Hardness Variation with Depth in an Open Pit Copper Deposit

Figure 15 – Example of Correlation Between Point Load Index (IS50) and DWi
Hardness Distributions

From SMC Testing’s data base it appears that the mean DWi, Mia, Mic and Mih values of deposits worldwide are normally distributed (see Figures 17-19). The same is not true for the A*b parameter which is highly skewed (Figure 20) and can be described by a Burr distribution (ref??)(Figure 21). Alternatively 1/(A*b) is found to be very nearly normally distributed (Figure 22). Although the distribution of the mean hardnesses of deposits is of general interest, of more importance is the distribution of hardness values within deposits as one of conditions for equation 8 to be applicable is that they are normal (or nearly normal). Figures 23 and 24 give an example of a deposit where it was found that the distribution of hardness values was nearly normal. This contrasts with the example shown in Figures 25 and 26 where the distribution was skewed. The reason for the skewness was found to be due to the fact that within the deposit there was at least two ore types with quite different hardnesses. In this particular deposit these ore types were easily differentiated via geological descriptions and when separated out were each found to be approximately normally distributed (see Figure 27). A further finding was that the coefficients of variation of the two ore types were much lower than that of the combined (skewed) distribution. It is likely, therefore, that in general if the coefficient of variation of hardness values is relatively high it signifies that there are at least two distinct hardness domains in the deposit which requires further investigation (and characterisation) to separate them out. This may help explain the bimodality of Figure 12 in that the upper modal group probably contains deposits with mixtures of ore types with significant differences in hardness whilst those in the lower group can be considered as comprising a single ore type having reasonably uniform hardness behavior. The expectation would be that this latter group would be normal or near normal in terms of distribution.
Figure 17 – Histogram of Mean DWi Values of Deposits in the SMC Test® Data Base

Figure 18 – Cumulative Frequency Distribution of Mean DWi Values of Deposits in the SMC Test® Data Base
Figure 19 - Cumulative Frequency Distribution of Mean Mia, Mic and Mih Values of Deposits in the SMC Test® Data Base

Figure 20 - Histogram of Mean A*b Values of Deposits in the SMC Test® Data Base
Figure 21 - Cumulative Frequency Distribution of Mean A*b Values of Deposits in the SMC Test® Data Base

Figure 22 - Cumulative Frequency Distribution of Mean 1/A*b Values of Deposits in the SMC Test® Data Base
Figure 23 – Example of Histogram of DWi Values in a Deposit (Approximately Normal)

Figure 24 - Example of Cumulative Distribution of DWi Values in a Deposit (Approximately Normal)
Figure 25 – Example of Histogram of DWi Values in a Deposit (Skewed)

Figure 26 - Example of Cumulative Distribution of DWi Values in a Deposit (Skewed)
CONCLUSIONS

The choice of appropriate laboratory tests for orebody hardness characterisation should be made on the basis of its relevance to the associated comminution circuit and demonstrated accuracy of the techniques/equations that subsequently use the test results to predict plant performance.

The precision of the test is also of importance. Independent studies have shown that the Bond crushing work index has very poor precision, whilst that of the rod mill test although better is still quite poor. In the case of Bonds ball mill work index test it has much better precision than the previous two. The JK drop-weight and SMC Tests have the best precision.

In terms of the required number of tests a staged approach is recommended in which, as projects develop from the scoping stage, knowledge of the variability of the deposit is progressively built and used to drive the number of tests required in subsequent stages.

Using classical statistics from variability data from over 1800 deposits, guidelines have been developed which should help indicate the number of samples that are required from the initial stages of design through to development of geometallurgical models for established plants.

ACKNOWLEDGEMENTS

The contribution of Andrew Morrell in assembling the SMC Test data base and providing the statistics associated with it is gratefully acknowledged. The insight of Aidan Giblett into the potential underlying causes of the shape of Figure 12 is also gratefully acknowledged.
REFERENCES


Morrell, S., 2009, Predicting the overall specific energy requirement of crushing, high pressure grinding roll and tumbling mill circuits, Minerals Engineering, Volume 22, Issue 6, May, Pages 544-549

Morrell, S., 2010, Predicting the specific energy required for size reduction of relatively coarse feeds in conventional crushers and high pressure grinding rolls, minerals engineering Volume 23, Issue 2, January, Pages 151-153


SME, 2019. Mineral Processing and Extractive Metallurgy Handbook by Robert C. Dunne (Editor), S. Komar Kawatra (Editor), Courtney A. Young (Editor)
